Lecture 11

Dynamic Asset Pricing Models - II

Fixing the C-CAPM

The risk-premium puzzle is a big drag on structural models, like the C-CAPM, which are loved by economists. A lot of efforts to salvage them:

- (1) Power utility function is too strict, since EIS and γ are unrealistically linked. Epstein-Zin-Weil's effort. Hansen, Heaton and Li (2007).
- (2) Model risk aversion parameter. A time-varying γ_t?. Constantinides's (1990) habit formation. Abel (1990), Campbell and Cochrane (1999).

- (3) Use different consumption goods: durables and non-durables. Eichenbaum and Hansen (1990), Pakos (2004), Yogo (2004).

- (4) Problem with $E_t[r_{t+1}]$. Ex-ante and ex-post are different. Soderlind's (2006) survey data. Peso problem? Rietz's (1988) disaster state. Also, add learning to model: Bayesian learning. Latest effort: Cogley and Sargent (2008). Survival bias? Brown, Goetzmann and Ross (1995).

Habit Formation

- *Constantinides* (1990): Average Joe's utility depends on the difference between current consumption and the "habit":

$$U(c) = E_t\, \Sigma_i\, \beta^i\, [(c_{t+i} - x_{t+i})^{1-\gamma}]/(1-\gamma), \qquad \lambda > 0.$$

Let the habit depend on lagged consumption (an *internal* habit):

$$x_{t+1} = \lambda\, c_{t+i-1}$$

$$U'(c_{t+i}) = E_{t+i}\ \{\beta^i\, (c_{t+i} - \lambda\, c_{t+i-1})^{-\gamma} + \beta^{i+1}\, (c_{t+i+1} - \lambda\, c_{t+i})^{-\gamma}\ (-\lambda)\}$$
$$= \beta^i\, c_{t+i-1}^{-\gamma}\, ((c_{t+i}/c_{t+i-1}) - \lambda)^{-\gamma} - \lambda\, \beta^{i+1}\, c_{t+i}^{-\gamma}\, E_{t+i}[((c_{t+i+1}/c_{t+i}) - \lambda)^{-\gamma}]$$
$$= \beta^i\, c_{t+i-1}^{-\gamma}\, \{(g_{t+i} - \lambda)^{-\gamma} - \lambda\beta g_{t+i}^{-\gamma}\ E_{t+i}[(g_{t+i+1} - \lambda)^{-\gamma}]\} \quad (g_{t+i} = c_{t+i}/c_{t+i-1})$$

If we assume that g_{t+i} is iid (not a realistic assumption):
$E_{t+i}[(g_{t+i+1} - \lambda)^{-\gamma}] = E_t[(g_{t+1} - \lambda)^{-\gamma}] = E[(g_t - \lambda)^{-\gamma}] = \Theta$ (a constant)

- Then, recall that $m_{t+1} = U'(c_{t+1})/ U'(c_t)$

$$m_{t+1} = \beta\, g_t^{-\gamma}\{(g_{t+1}-\lambda)^{-\gamma} - \lambda\, \beta\, g_{t+1}^{-\gamma}\, \Theta\}/\{(g_t-\lambda)^{-\gamma} - \lambda\, \beta\, g_t^{-\gamma}\, \Theta\}$$

Compare this to m_{t+1} from the standard model:

$$m_{t+1} = \beta\, g_{t+1}^{-\gamma}$$

- Now, since $\lambda>0$, and $(g_{t+1}-\lambda) < 1$ is more often than $g_{t+1} < 1$. Thus, we do not need a large risk-aversion coefficient to amplify the variation of consumption.

Note: If $\lambda =1$ and the habit is fixed, the utility function becomes:

$U(c) = E_t\, \Sigma_i\, \beta^i\, [(c_{t+i} - x)^{1-\gamma}]/(1-\gamma)$ x: fixed subsistence level (habit).
$RRA = \gamma /(1 - x/c_t)$ (if $x/c_t=0.8$, $RRA= 5\, \gamma$.)

- But, risk aversion is now time-varying. This utility function makes the agent very averse to consumption risk. It helps to address the low risk-free rate.

• *Campbell and Cochrane's* (1999): Take on Abel's (1990) formulation of habit formation. Let U(c) be

$$U(c) = E_t\, \Sigma_i\, \beta^i\, [(C_{t+i} - X_{t+i})^{1-\gamma} - 1\,]/(1-\gamma),$$

where x_t is the level of habit.

CC work with "surplus consumption ratio" –their *state* variable:

$$S_t = (C_t - X_t)/C_t$$

CC define the habit x_t as *external*: Average Joe looks at aggregate consumption (the "Joneses") to determine his level of happiness:

$$S^a_t = [(C^a_t - X_t)/C^a_t$$

Now, $m_{t+1} = \beta[(C_{t+1}/C_t)(S_{t+1}/S_t)]^{-\gamma}$

CC assume a $s_t= \log(S_t)$ follows an heteroscedastic AR(1) process:

$$s^a_t = (1-\varphi)\, s + \varphi\, s^a_{t-1} + \lambda(s^a_t)\, (c^a_t - c^a_{t-1} - g)$$

CC assume $\Delta c_t=\log(C_t/C_{t-1}) \sim$ iid lognormal (g,σ^2).

Now, the risk-free rate:

$$r_f = -\ln(\beta) + \gamma g - \gamma(1-\varphi)(s_t - s) - \gamma^2\sigma^2/2\,[1+\lambda(s_t)]^2$$

CC have to choose $\lambda(s_t)$ to proceed. They use a simple threshold function –i.e., with two (implicit) surplus states.

• CC explain the volatility puzzle and the low interest rate puzzle. But they also need a high risk aversion coefficient to match the equity premium (average risk aversion 80, in low surplus state gets to 100!).

• CC find that agents fear stocks primarily because they do badly in recessions (times of low surplus consumption ratios), not because stock returns are correlated with declines in wealth or consumption.

• CC conclude that habit formation, or some other device is needed to generate time-varying countercyclical risk premia along with relatively constant risk-free rates.

Separating EIS (ψ) and γ

- *Epstein and Zin* (1991) – *Weil* (1989): They work with a recursive utility function:

  $$U_t = \{(1-\delta)\, C_t^{(1-\gamma)/\theta} + \delta\, (E_t[\,U_{t+1}^{(1-\gamma)}\,]^{1/\theta}\}^{\theta/(1-\gamma)}$$

  where $\theta = (1-\gamma)/(1-1/\psi)$. When $\theta=1$, the recursion becomes linear (usual power utility model).

- The intertemporal budget constraint is

  $$W_{t+1} = (1+R_{m,t+1})\,(W_t - C_t) \qquad (W_t = \text{Average Joe's wealth.})$$

- Assuming lognormality and homoscedasticity:

  $$r_{f,t+1} = -\ln\delta + (\theta-1)/2\,\sigma_m^2 - \theta/(2\psi^2)\,\sigma_c^2 + 1/\psi\, E_t[\ln(c_{t+1}) - \ln(c_t)]$$

• Then, the risk premium for asset i is:

$$E[r_{i,t+1}] - r_{f,t+1} = \theta/\psi \, \sigma_{ic} - (1-\theta)\, \sigma_{i,m} - \sigma_c^2/2$$

The Epstein-Zin-Weil model nests the static CAPM (θ=0) and the C-CAPM with power utility ((θ=1).

• Campbell (1993) introduces a log-linearized budget constraint and after a lot of substitutions, we get:

$$E[r_{i,t+1}] - r_{f,t+1} = \gamma \, \sigma_{im} + (1-\gamma)\, \sigma_{ih} - \sigma_i^2/2,$$

where σ_{ih} is the covariance of asset i with "news" about future returns on the market –see CLM.

• Static CAPM?
- γ = 1 (no realistic)
- σ_{ih} = 0 (no realistic)
- $R_{m,t}$ follows a univariate process, then future returns are perfectly correlated with current return. (Maybe?).

• Findings: Equity premium estimates using this model are lower γ. But, with an unrealistic consumption volatility.

More General Utility Functions

- Introduce nonseparabilities: consumption and some other good.

- Usually done using Cobb-Douglas utility functions. Then, we have an easy to work marginal utility function:

$$u'(C_t,X_t) = C_t^{-\gamma 1} X_t^{-\gamma 2}$$

where X_t represents some other good.

- Now, we have an easy to work Euler's equation:

$$1= E_t[\beta(C_{t+1}/C_t)^{-\gamma 1}(X_{t+1}/X_t)^{-\gamma 2} (1+R_{t+1}].$$

Assuming joint lognormality and homoscedasticity:

$$E_t[r_{i,t+1}] = \mu_i + \gamma_1 E_t[\Delta c_{t+1}] + \gamma_2 E_t[\Delta x_{t+1}] ,$$

Other goods: leisure -Eichenbaum, Hansen and Singleton (1988)
government spending – Aschauer (1985)
stock of durable goods – Startz (1989)

Findings: None of the X_t variables significantly improves the CAPM.

- Surprised? Not really. The proposed X_t variables do not have enough variability to explain the variability of excess returns.

Ex-ante ≠ Ex-post: Survey Evidence

- C-CAPM: $E[r_t] - r_{f,t} = Cov(r_t, \Delta c_t)\ \gamma =$
  $= Corr(r_t, \Delta c_t)\ \sigma_c\ \sigma_m\ \gamma$

- Fitting U.S. ex-post data (1952–2005) creates the "EP puzzle:"

$E[r_t] - r_{f,t} = .06 = Corr(r_t, \Delta c_t)\ \sigma_c\ \sigma_m\ \gamma = 0.17$ (or 0.33) x 0.14 x 0.01 x γ

=> Requires risk aversion, γ, to be very high: $\gamma \geq 43$!

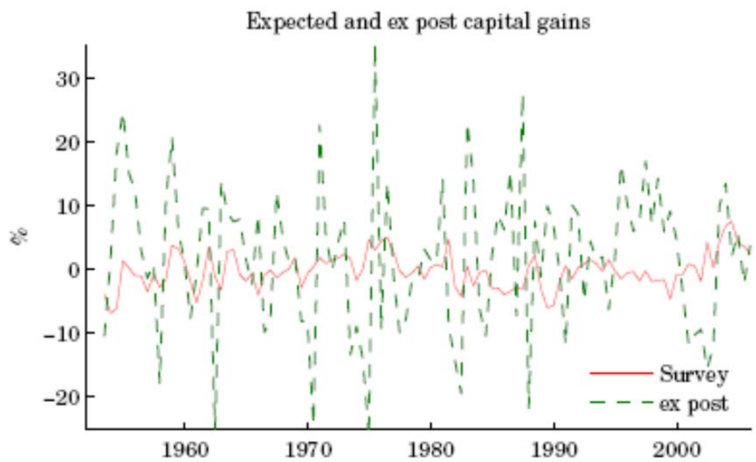
- Soderling (2006) tests the C-CAPM using ex-ante data:
  - For $E[r_t] - r_{f,t}$, survey data (Livingston survery (1952-2005)
  - For σ_c, survey data (SPF: survey of professional forecasters)
  - For σ_m, no survey data. Expectations extracted from S&P options.
  - For $Corr(r_t, \Delta c_t)$, no expectations data.

- This approach avoids the usual "joint test:" C-CAPM and RE.

- Data:
  – Livingston survey: survey of economic experts (1952–2005). Before 1990: S&P Industrials; After 1990, S&P 500 Composite.
  – SPF: survey of economic experts about probabilities of GDP growth rates –assume results carry over to consumption growth.
  – Lots of S&P 500 options (daily data 1985-2005).

- *Example*: Excess returns survey

	1952–1990	1990–2005	1952–2005
Survey expectations	−1.1	1.5	−0.4
S&P Industrials, ex post	3.2		
S&P 500, ex post		5.3	
S&P combined, ex post			3.8
Dividend yield S&P 500	4.0	2.1	3.5



• $E[r_t] - r_{f,t} \approx 3\%$ - 3.5% (about half the historical mean excess return)

• Volatile market, hard to "learn." Test for zero average error.
Average forecast error = 3%, SD of forecast error = 16%, T=100
=> $t = 3/(16/100^{1/2}) = 1.9$

• Ex-ante estimates:
- $E[r_t] - r_{f,t} \approx 0.5 \times 0.06 = .03$
- $\sigma_c \approx 0.6 \times 0.01 = .006$
- $\sigma_m, \approx 1.15 \times 0.14 = .161$
- $\mathrm{Corr}(r_t, \Delta c_t) = ?$.

$E[r_t] - r_{f,t} = .03 = \mathrm{Corr}(r_t, \Delta c_t)\ \sigma_c\ \sigma_m\ \gamma = 0.17$ (or 0.33) $\times 0.161 \times 0.006 \times \gamma$

=> Now, γ needs to be 0.7 of γ in ex-post data (30 instead of 43)

• Better for C-CAPM, but still a very high γ

Learning

- *Cogley and Sargent* (2008): Follow Friedman and Schwartz (1963), where the U.S. 1930 depression created a pessimist mood, seriously affected estimates of expected returns. (The "depression generation.")
- Siegel (1992) presents supporting evidence: 1802-1925 Equity premium was 2.0%; 1926-1990 Equity premium was 5.9%.

- CS present a learning model to explain the equity premium.
- Assumptions:
  - Consumption is driven by a two-state Markov process.
  - Representative agent is a Bayesian learner.
  - Initial beliefs from the 1930s are very pessimistic.
  - Learning is slow.
  - Asset pricing is distorted by these beliefs for a long time.

• **Related work**: Cecchetti, Lam, Mark (2000).
Equity premium due to distorted beliefs. CLM (2000) have no learning. Agents are naturally and permanently pessimistic.

• Under Euler's equation we have:

$$p_t = E_t^s [m_{t+1}\ x_{t+1}]$$

If E_t^s is the expectation implied by the true transition probabilities F, the agent has rational expectations. Call this E_t^a. The equity premium will be small in that case.

CLM (2000) show that $E_t^s \neq E_t^a$ explain equity premium. But, we have permanently distorted beliefs.

CLM (2000) pessimistic views come from agents assigning a larger probability to the bad ("depression") state.

CS model consumption as:

$$\Delta\ln(C_t) = \mu(S_t) + \varepsilon_t,$$

where S_t is a state (H, L) variable. S_t moves according to the transition matrix F, with elements F_{ij} = Prob[S_t=j|S_{t-1}=i] -a Hamilton process.

Table 1: Maximum Likelihood Estimates of the Consumption Process

	F_{hh}	F_{ll}	μ_h	μ_l	σ_ε
Estimate	0.978	0.515	2.251	-6.785	3.127
Standard Error	0.019	0.264	0.328	1.885	0.241

Note: Reproduced from Cecchetti, et. al. (2000)

Note: F_{ll}, the probability that a contraction will continue is estimated at 0.515 with a standard error of 0.264. A 90% C.I.: [.079, 0.951], which implies that contractions could have a median durations ranging from 3 months to 13 years.

=> even with 100 years of data, big model uncertainty persists.

• **Back to CS**

• CS allows Average Joe to learn their way out of their pessimism.

• Simplify model by setting ε_t=0 (simpler learning problem), but assume:

$$g_t = 1+ \mu(h)/100 \qquad \text{if } S_t\text{=H (=1)}$$
$$= 1+ \mu(l)/100 \qquad \text{if } S_t\text{=L (=0)}$$

• CS also take CLM's μ(h) and μ(l) –known by the agents- and F_{ll} and F_{hh} –unknown. Average Joe applies Bayes's theorem to learn F_{ij}.

• Average Joe uses a beta-binomial probability model for learning about $\Delta\ln(C_t)$. A binomial likelihood is a good candidate, a beta density is the conjugate prior for a binomial likelihood.

• Average Joe has independent beta priors over (F_{hh},F_{ll}) .

• Average Joe counts the number of transitions from state i to state j (n_t^{ij}) through date t and learns. The agent has a prior (n_0^{ij}).

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• Average Joe counts the number of transitions from state i to state j (n_t^{ij}) through date t and learns. The agent has a prior (n_0^{ij}). Then:

$$p(F_{hh}, F_{ll}) = p(F_{hh})p(F_{ll}),$$

where

$$p(F_{hh}) \propto F_{hh}{}^{n_0^{hh}-1}(1-F_{hh})^{n_0^{hl}-1},$$
$$p(F_{ll}) \propto F_{ll}{}^{n_0^{ll}-1}(1-F_{ll})^{n_0^{lh}-1}.$$

• Average Joe counts the number of transitions from state i to state j (n_t^{ij}) through date t and learns. The agent has a prior (n_0^{ij}).

• The updating (learning) rule:

$n_{t+1}^{ij} = n_t^{ij} + 1$ if s_{t+1}=j and s_t=i,

$n_{t+1}^{ij} = n_t^{ij}$ otherwise.

• The date-t estimate of F is formed from the counters:

$$F_t = \begin{bmatrix} \frac{n_t^{hh}}{n_t^{hh}+n_t^{hl}} & \frac{n_t^{hl}}{n_t^{hh}+n_t^{hl}} \\ \frac{n_t^{lh}}{n_t^{lh}+n_t^{ll}} & \frac{n_t^{ll}}{n_t^{lh}+n_t^{ll}} \end{bmatrix}.$$

• The agent makes decisions based on the values in this F matrix. The values are treated as constants when making decisions, but are random variables until convergence to RE equilibrium.

• CS need a starting point, where agents are "pessimistic" (or with "shattered" beliefs, because of the depression). Think of a "worst case" F=F^{WC}. (The WC model should be hard to reject in a training sample T_0.)

<u>Note</u>: This is partial equilibrium, as many in the asset pricing literature. The agent cannot affect the system by changing beliefs. Hence, there is no active learning incentive. Agents "wait." Learn about F_{hl} complicated.

• **Estimation**
• CS draw 1, 000 consumption growth paths of 70 years each, assuming the true Markov chain given by F and μ(h) and μ(l).

• Pessimistic Average Joe is endowed with F^{WC}. Then, he determines asset prices using Euler's equation and applies Bayes rule each period.

$$P_t(S_t=i,C_t) = \beta\ \Sigma_j\ F_{ij}(t)\ (g_{j,t+1})^{-\gamma}\ [P_{t+1}(S_{t+1}=j,\ g_{j,t+1}C_t) + g_{j,t+1}C_t]$$

Table 4: The Mean, Standard Deviation, and Sharpe Ratio for Excess Returns

	1872-2002	1872-1928	1929-2002	1929-1965	1966-2002
$E(R_{xt})$	0.0410	0.0266	0.0521	0.0708	0.0334
$\sigma(R_{xt})$	0.1734	0.1507	0.1892	0.2239	0.1474
$E(R_{xt})/\sigma(R_{xt})$	0.2364	0.1765	0.2754	0.3162	0.2266

Table 5: Model Sharpe Ratios

	$T_0 = 10$	30	50	70
Full Simulation	0.301	0.394	0.378	0.357
First Half	0.443	0.531	0.473	0.436
Second Half	0.190	0.272	0.286	0.281

Table 6: Model Equity Premia

	$T_0 = 10$	30	50	70
Full Simulation	0.0536	0.0333	0.0258	0.0216
First Half	0.0702	0.0431	0.0321	0.0263
Second Half	0.0370	0.0236	0.0195	0.0169

Findings: Two anomalies are explained

(1) High market premium, but low risk aversion (from surveys) –i.e, the equity premium.

(2) Ex-post arbitrage opportunities: Euler equations hold ex-ante, but with respect to the agents' subjective F, not the ex-post realized frequencies.

Note: Slow convergence is critical to explain anomalies.

Survival

- *Ex post (*observed) $E_t[r_{t+1} - r_f] >$ *ex ante* $E_t[r_{t+1} - r_f]$
- *Ex post* $E_t[r_{t+1} - r_f]$ decreases with crash frequency
- Cross-sectional implications
  - Low risk markets have higher *ex post* premium

- International evidence: From Goetzmann and Jorion (1999)
  - U.S. market shows the maximum equity risk premium (5.48%)
  - 6 of 21 markets experienced no interruption from the 1920's.
  - 8 had a temporary closure and 7 suffered a long-term closure.
  - The "Non-U.S. Survived markets" returned 4.52%. while the "Non-U.S. All markets" returned 3.84%.
  - Survival bias is around .60%
  - The GJ evidence points more towards a "good draw" for the U.S.